IPP-QM-12: The PBR theorem

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MT25

The course

1. Basic quantum formalism
2. Density operators and entanglement
3. Decoherence
4. The measurement problem
5. Dynamical collapse theories
6. Bohmian mechanics
7. Everettian structure
8. Everettian probability
9. EPR and Bell's theorem
10. The Bell-CHSH inequalities and possible responses
11. Contextuality
12. The PBR theorem
13. Quantum logic
14. QBism
15. Pragmatism and relational quantum mechanics
16. Wavefunction realism

Today

The ontological models framework

Introducing Ψ-epistemic approaches

The PBR theorem

Hardy's ontic state indifference theorem

The BCLM inequality

Whither Ψ-epistemic?

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- Probability distribution $\epsilon_M(q|\lambda)$, which is the probability distribution of getting some outcome *q* in measurement *M* given that the ontic state is λ.

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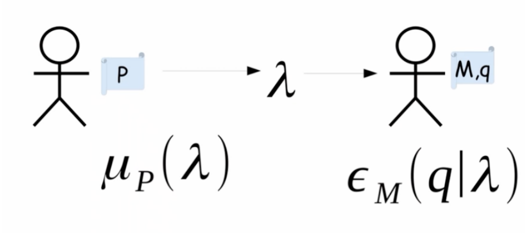

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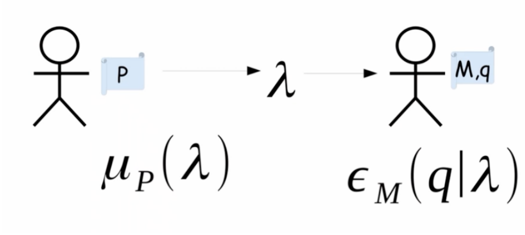

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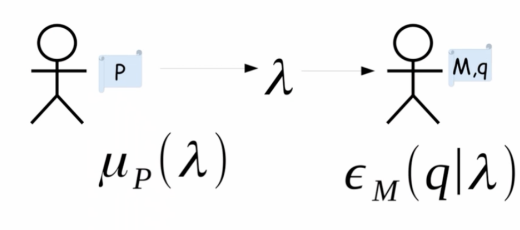

- The preparation P gives some probability distribution $\mu_P(\lambda)$ over ontic states λ.
- For each of these ontic states, there is some probability distribution $\epsilon_M(q|\lambda)$ over outcomes q in measurement M.

Ontological models and operational models

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- The LHS is a purely operational thing; it has been purged of any mention of ontic states!
- In standard quantum theory (where the preparation *just is* the initial quantum state), we of course have

$$F(q|M,P) = |\langle q|P\rangle|^2,$$

i.e. the Born rule.

The inevitability of ontological models

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- Provably, for any operational model $F(q|M, P)$, there is some ontological model compatible with it. (Basically just interpolation.)
- Since ontological models help us to explain the operational outcomes, why not take them seriously?

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- Roughly, according to these positions, there is *only* 'extra structure'; there is no wavefunction.
- 'Ψ-epistemic' approaches are *not* anti-realist, because they seek to model and study this 'extra structure'.
- (Rather, they're just anti-realist about wavefunction, hence 'Ψ-epistemic': at best, the quantum state Ψ encodes our ignorance; it is not something real.)

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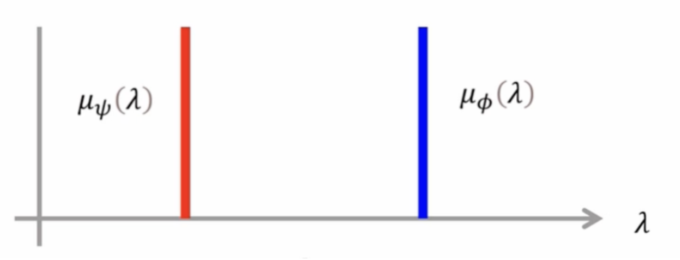
Ask: must the quantum state ψ be part of the ontic state λ?

Yes: 'Ψ-ontic'. (And if $\lambda = \psi$, then 'Ψ-complete'.)

No: 'Ψ-epistemic'.

Ψ-complete

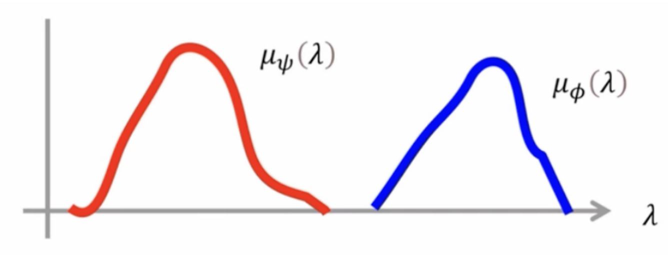
Knowing the quantum state ψ nails down the ontic state λ:



So: $\lambda \Leftrightarrow \psi$.

Ψ-ontic

The ontic state λ implies the quantum state ψ but might contain other things as well.



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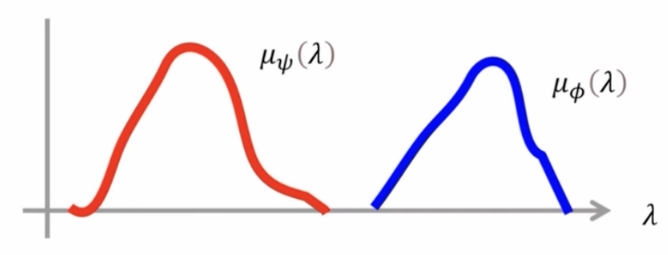

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(Bohmians live here! Even for the Bohmians who want to treat Ψ as 'nomological', etc.)

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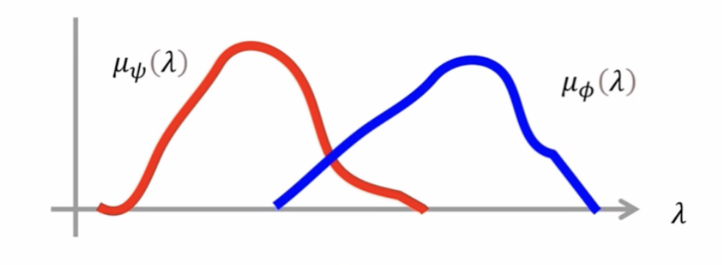

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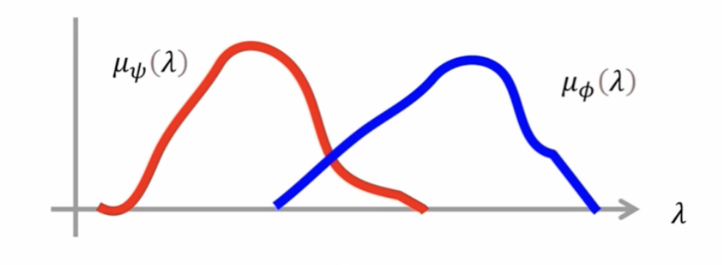

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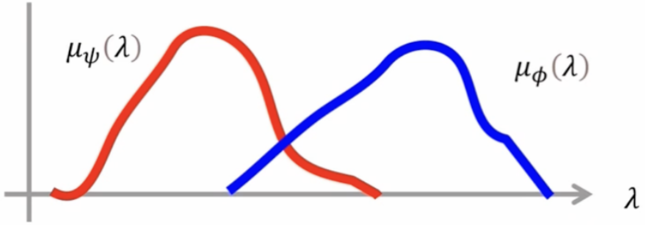

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Surprisingly, it took until circa 2012 for people to come up with Ψ-epistemic models! But they are now a focus of significant investigation and study.

Challenges for Ψ-epistemic approaches

But are Ψ-epistemic approaches actually viable? In the remainder of this lecture, we're going to be looking at the following results, which cast this into doubt:

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1. The Pusey–Barrett–Rudolph (PBR) theorem (2012)
2. Hardy's ontic state indifference theorem (2013)
3. The Barrett–Cavalcanti–Lal–Maroney (BCLM) inequality (2014)

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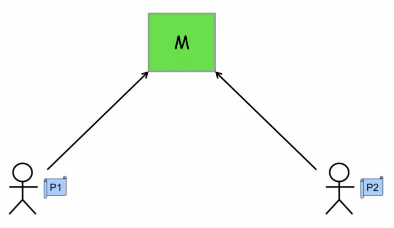

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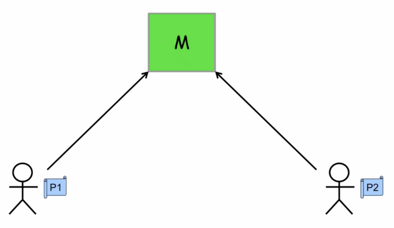

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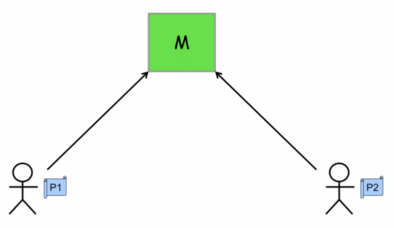

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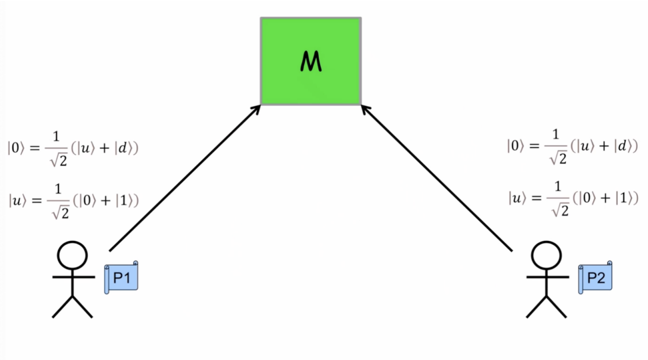
- (Somewhat akin to the opposite of the Bell scenario—two spatially separated *preparers* sending their prepared states to a joint *future* where they are measured jointly.)

The PBR theorem

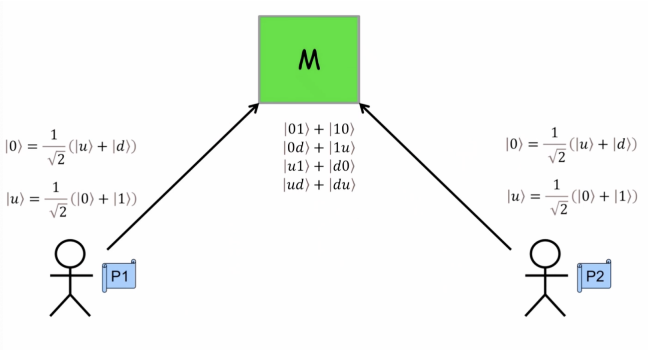
Alice and Bob prepare non-orthogonal states:



They prepare their states, send them off, and will later compare outcomes.

The PBR theorem

The measuring procedure measures this joint entangled basis:



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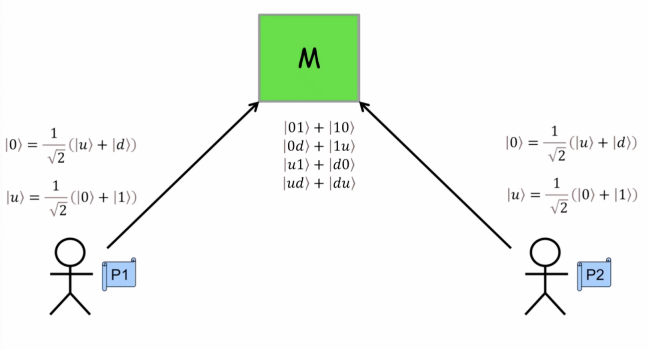

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(**Exercise:** Confirm this, by writing out the states in the $\{|0\rangle, |1\rangle\}$ basis.)

The PBR theorem

We now have the following table of inner products:

.	$\|00\rangle$	$\|0u\rangle$	$\|u0\rangle$	$\|uu\rangle$
$\|01\rangle + \|10\rangle$	0	1/4	1/4	1/2
$\|0d\rangle + \|1u\rangle$	1/4	0	1/2	1/4
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From this we see that: (Here, Λ_α is the space of possible ontic states λ consistent with the quantum state $|\alpha\rangle$.)

$$\begin{aligned}
&\forall\lambda \in \Lambda_{00}, && \epsilon_M(01 + 10|\lambda) = 0\\
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&\forall\lambda \in \Lambda_{u0}, && \epsilon_M(u1 + d0|\lambda) = 0\\
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\end{aligned}$$

(Zero probabilities of certain outcomes given where the ontic state λ is located in state space.)

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- No intersection between the state spaces is equivalent to saying $\mu_{00}(\lambda)\mu_{0u}(\lambda)\mu_{u0}(\lambda)\mu_{uu}(\lambda) = 0$.

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- This means, for both λ_1 and λ_2, that there is *no overlap* in the probability distributions of the $|0\rangle$ or $|u\rangle$ preparations.
- *But this just means that we are working with a* Ψ*-ontic model!*

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- How bad is the failure of preparation independence?
- However one cuts it, the failure seems to be a kind of non-locality.

Possible responses to the PBR theorem

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   - How to recover local ontic properties measured in experiments?

Assessing the possible responses to the PBR theorem

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Note that there are parallels here with the various responses to Bell's theorem (Lectures 9 and 10).

Today

The ontological models framework

Introducing Ψ-epistemic approaches

The PBR theorem

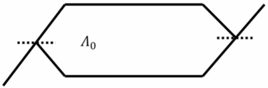
Hardy's ontic state indifference theorem

The BCLM inequality

Whither Ψ-epistemic?

Hardy's ontic state indifference theorem

Consider an interferometer:



$$|0\rangle = \frac{1}{\sqrt{2}}(|u\rangle + |d\rangle)$$

Hardy's ontic state indifference theorem

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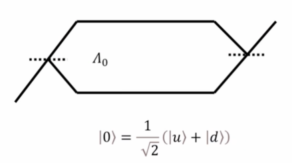

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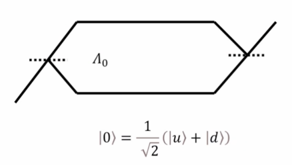

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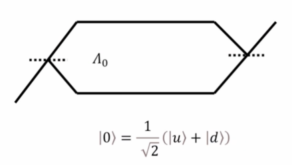
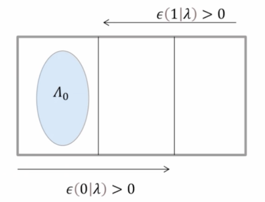
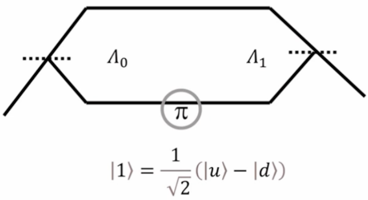


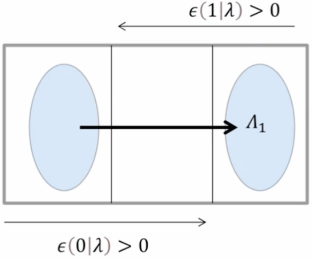
- Consider the $|0\rangle$ state as above.
- Consider the ontic state space for the $|0\rangle$ preparation, Λ_0.
- For this preparation, the system couldn't have been in the region of state space where the outcome could have been recorded as 1, so we have:



Hardy's ontic state indifference theorem

If I had inserted a phase shifter π in the lower half of the interferometer, I would have shifted the entire ontic state to being somewhere in Λ_1:





A mass exodus of ontic states...

Hardy's ontic state indifference theorem

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Now consider the up state $|u\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ with ontic state space Λ_u.

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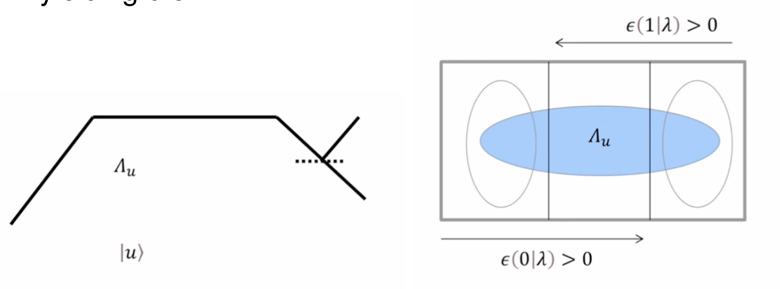
- When it hits the beam splitter, it has a 50/50 chance of yielding 0 or 1:



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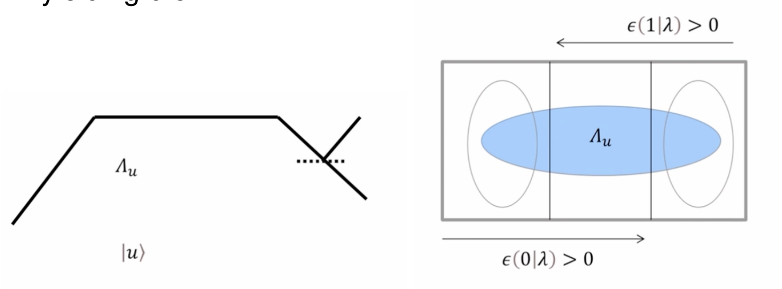
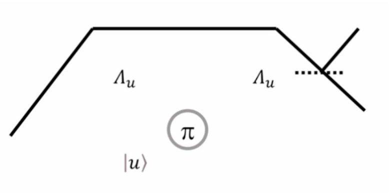
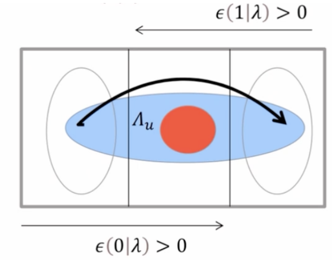


Here, that Λ_u extends into the left-hand box and right-hand box makes the model Ψ-epistemic, because these are, respectively, the intersection with the Λ_0 and Λ_1 (recall the original diagram illustrating Ψ-epistemic theories.)

Hardy's ontic state indifference theorem

Now do the phase shift in the branch of the interferometer where the wavepacket never goes:





Some ontic states (those in the left-hand box) have to be shifted over even though the interferometer acted where the wavepacket never goes (*mutatis mutandis* the other way). ('Remote invasiveness'.)

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- Given this assumption ('ontic state indifference'), the ontic states can only lie in the red region.
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- The reason being that now Λ_u has no intersection with either Λ_0 or Λ_1. (Recall again the original diagram illustrating Ψ-epistemic theories.)

How bad is remote invasiveness?

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- Ontic state affected by operations performed at remote locations!
- Almost any region which is causally connected with a common past and common future!
- So operations performed almost anywhere can affect the local ontic properties!

Today

The ontological models framework

Introducing Ψ-epistemic approaches

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Whither Ψ-epistemic?

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- (I won't trouble you with a proof or an explicit statement of the result, both of which would take us a bit too far afield, but see the original (short!) paper for the details.)

Three inequalities for the three no-go theorems

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Exercise: Compare the conceptual status of these three inequalities with respect to their respective no-go theorems. Which invite us to be doing 'experimental metaphysics'?

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- Reject the game: “The wavefunction represents neither part of an ontic state, nor a probability distribution over such states”. Direction of general anti-realism?

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Next week: quantum logic and QBism.

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